Department of Mathematics
University of Notre Dame
Math 10120 – Finite Math
Spring 2019

Name:______________________________

Instructor: Juan Migliore

Exam 2

March 7, 2019

This exam is in two parts on 10 pages and contains 15 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. You may use a calculator, but no books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached.

You must record on this page your answers to the multiple choice problems.

The partial credit problems should be answered on the page where the problem is given. The spaces on the bottom right part of this page are for me to record your grades, **not** for you to write your answers.

Place an $\times$ through your answer to each problem.

1.	(a)	(b)	(c)	(d)	(e)
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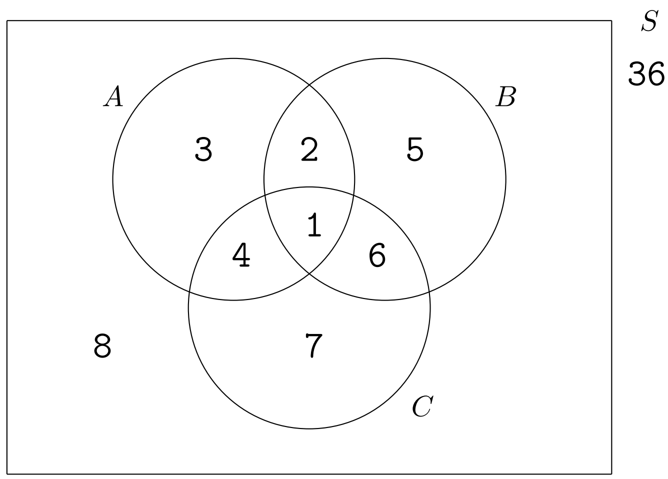
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)

MC.	______
11.	______
12.	______
13.	______
14.	______
15.	______
Tot.	______

Multiple Choice

1. (5 pts.) The 36 members of the Travel Club have upcoming trips planned to Argentina (A), Bolivia (B) and Colombia (C). The following Venn diagram indicates how many members of the club are planning to go on the corresponding trips. If a student is chosen at random, what is the probability that she is going to Argentina?



(a) $\frac{3}{36}$ ~~(b)~~ $\frac{10}{36}$ (c) $\frac{9}{36}$ (d) $\frac{3}{10}$ (e) $\frac{1}{10}$

$$P(A) = \frac{3+2+1+4}{36} = \frac{10}{36}$$

2. (5 pts.) Refer to the club and Venn diagram in problem #1. A student is chosen at random, and it is discovered that he is going to Argentina. With this extra information, what is the probability that he is also going to Bolivia?

~~(a)~~ $\frac{3}{10}$ (b) $\frac{2}{10}$ (c) $\frac{3}{14}$ (d) $\frac{2}{14}$ (e) $\frac{14}{36}$

$$P(B|A) = \frac{2+1}{3+2+1+4} = \frac{3}{10}$$

3. (5 pts.) Claire rolls a red die and a blue die and observes the sum. Find the probability that the sum is odd. [Note: "odd" means, in this case, either 3, 5, 7, 9 or 11.]

(a) $\frac{12}{36} = \frac{1}{3}$ (b) $\frac{9}{36} = \frac{1}{4}$ (c) $\frac{5}{11}$

(d) $\frac{6}{36} = \frac{1}{6}$ (e) $\frac{18}{36} = \frac{1}{2}$

(1,1) (1,2) (1,3) (1,4) (1,5) (1,6)
(2,1) (2,2) (2,3) (2,4) (2,5) (2,6)
(3,1) (3,2) (3,3) (3,4) (3,5) (3,6)
(4,1) (4,2) (4,3) (4,4) (4,5) (4,6)
(5,1) (5,2) (5,3) (5,4) (5,5) (5,6)
(6,1) (6,2) (6,3) (6,4) (6,5) (6,6)

$\frac{18}{36} = \frac{1}{2}$

4. (5 pts.) Emily flips a coin 6 times. Find the probability that the coin shows Heads exactly 3 times. Give your answer as a fraction in lowest terms.

(a) $\frac{1}{2}$ (b) $\frac{1}{8}$ (c) $\frac{5}{16}$ (d) $\frac{1}{20}$ (e) $\frac{3}{16}$

$\frac{C(6,3)}{2^6} = \frac{20}{64} = \frac{5}{16}$

5. (5 pts.) In a certain town in Canada, 50% of the population speaks English, 70% speaks French and 10% doesn't speak either English or French. A person is chosen at random. What is the probability that she speaks both English and French?

(a) 90% (b) 10% (c) 20% ~~(d)~~ 30% (e) 60%

$P(E \cup F) = P(E) + P(F) - P(E \cap F)$

We know 10% don't speak either English or French, so 90% speak at least one, i.e. $P(E \cup F) = .90$

Then $.90 = .50 + .70 - P(E \cap F)$, so $P(E \cap F) = .5 + .7 - .9 = .30$

6. (5 pts.) The Smith family has 10 children, consisting of four boys and six girls. After a big snowstorm, the parents randomly choose three of the children to shovel the driveway. What is the probability that **at least** one of the three is a boy?

~~(a)~~ $\frac{5}{6}$ (b) $\frac{1}{6}$ (c) $\frac{1}{3}$ (d) $\frac{1}{2}$ (e) $\frac{2}{3}$

Complement rule

$1 - P(\text{none is a boy}) = 1 - \frac{C(6,3)}{C(10,3)} = 1 - \frac{20}{120} = \frac{5}{6}$

7. (5 pts.) In a certain city, 55% of the people are male and 45% are female. Of the males, 15% have a certain disorder. Of the females, 5% have the disorder. A person is chosen at random and found **not** to have the disorder. What is the probability that **that** person is female? [Hint: start with a tree diagram.]

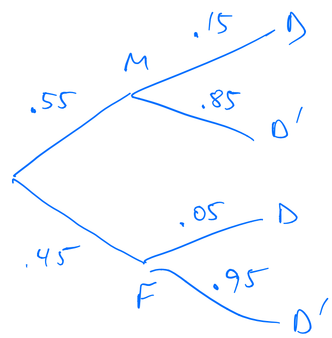
(a) $\dfrac{(.45)(.95)}{(.45)(.95)+(.45)(.05)}$ (b) $\dfrac{(.45)(.95)}{(.55)(.85)}$ (c) .45

(d) $(.45)(.95)$ ~~(e)~~ $\dfrac{(.45)(.95)}{(.45)(.95)+(.55)(.85)}$

M = male

F = female

D = disorder



$$P(F \mid D') = \frac{P(F \cap D')}{P(D')}$$

$$= \frac{(.45)(.95)}{(.45)(.95) + (.55)(.85)}$$

8. (5 pts.) In randomly chosen group of 15 people, what is the probability that **at least** two have the same birthday?

(a) $\dfrac{C(15,2)}{365^{15}}$ (b) $\dfrac{C(15,2)(363)(362)\cdots(351)}{365^{15}}$

~~(c)~~ $1-\dfrac{(365)(364)\cdots(351)}{365^{15}}$ (d) $1-\dfrac{C(15,2)}{365^{15}}$

(e) $1-\dfrac{C(15,2)\cdot C(363,2)}{365^{15}}$

Complement rule

$$1 - P(\text{all different}) = 1 - \frac{(365)(364)\cdots(351)}{365^{15}}$$

9. (5 pts.) I have a deck of 24 cards consisting of the A, 2, 3, 4, 5 and 6 of each suit. Bill, Claire, Doug and Emily are seated around a table. From this deck the dealer gives 2 cards to Bill, two cards to Claire, two cards to Doug and two cards to Emily (**without replacement**). What is the probability that **none** of them gets an ace? [Hint: of the 24 cards, four are aces and 20 aren't.]

~~(a)~~ $\dfrac{P(20,2) \cdot P(18,2) \cdot P(16,2) \cdot P(14,2)}{P(24,8)}$ (b) $1 - \dfrac{P(20,8)}{P(24,8)}$

(c) $\dfrac{P(24,2) \cdot P(22,2) \cdot P(20,2) \cdot P(18,2)}{P(24,8)}$ (d) $1 - \dfrac{4 \cdot P(20,2)}{P(24,8)}$

(e) $\dfrac{4 \cdot P(20,2)}{P(24,8)}$

$$\frac{(20)(19)(18)(17)(16)(15)(14)(13)}{(24)(23)(22)(21)(20)(19)(18)(17)} = \frac{P(20,8)}{P(24,8)} = \frac{P(20,2)\,P(18,2)\,P(16,2)\,P(14,2)}{P(24,8)}$$

10. (5 pts.) You are dealt 13 cards from a standard deck of 52 cards (without replacement). What is the probability that you get both the Jack of diamonds and the Queen of spades?

(a) $\dfrac{C(52,11)}{C(52,13)}$ (b) $\dfrac{P(50,11)}{C(52,13)}$ (c) $\dfrac{C(13,2) \cdot C(39,11)}{C(52,13)}$

~~(d)~~ $\dfrac{C(50,11)}{C(52,13)}$ (e) $\dfrac{2 \cdot C(50,11)}{C(52,13)}$

$$\frac{C(50,11)}{C(52,13)}$$

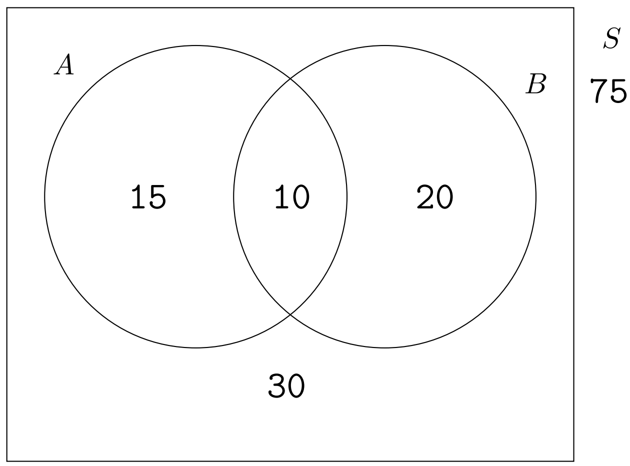
all ways of getting 13 cards

For the numerator you have the jack of diamonds and the queen of spades, and need 11 more from the remaining 50 cards

Partial Credit

You must show all of your work on the partial credit problems to receive credit! Make sure that your answer is clearly indicated. You're more likely to get partial credit for a wrong answer if you explain your reasoning.

11. (10 pts.)
The following Venn diagram describes the relative sizes of events A, B and C in a sample space S.



(a) Show that events A and B are independent. You have to fully explain your answer to get credit.

$$P(A) = \frac{15+10}{75} = \frac{1}{3}$$

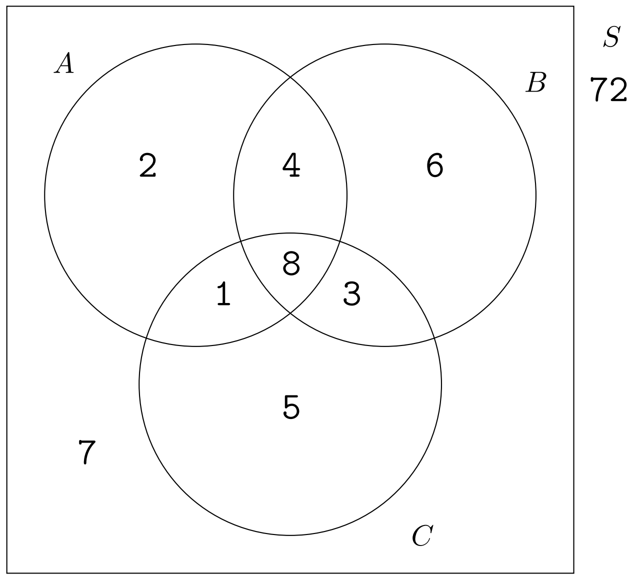
$$P(A|B) = \frac{10}{10+20} = \frac{1}{3}$$

Since $P(A) = P(A|B)$, A and B are independent

(b) Are events A and B mutually exclusive? Give a clear explanation of your answer.

No – Mutually exclusive means they have no elements in common, but A and B have 10 elements in common.

12. (10 pts.) The following Venn diagram describes the relative sizes of events A, B and C in a sample space S.



Find each of the probabilities using the numbers in the diagram. For example, if I asked for $P(A)$, I'd like you to write

$$\frac{2+4+8+1}{72}. \qquad \text{You would not have to write} = \frac{15}{72} = \frac{5}{24} \text{ unless you want to.}$$

(Any order of the numbers in the numerator would be fine.)

(a) $P(A \cap B) = \frac{4+8}{72} = \frac{12}{72} = \frac{1}{6}$

(b) $P(C \mid A) = \frac{1+8}{1+8+2+4} = \frac{9}{15} = \frac{3}{5}$

(c) $P(A \cap B \cap C') = \frac{4}{72} = \frac{1}{18}$

(d) $P(A \cap B \mid C) = \frac{8}{1+8+3+5} = \frac{8}{17}$

(e) $P(C \mid A \cap B) = \frac{8}{4+8} = \frac{8}{12} = \frac{2}{3}$

13. (10 pts.) (These numbers are totally fictitious.) In the city of Philadelphia, 99% of the people are Eagles fans and 1% are not.

- Of the Eagles fans, 98% own an Eagles sweatshirt and 2% do not.
- Of the non-Eagles fans, 5% own an Eagles sweatshirt and 95% do not.

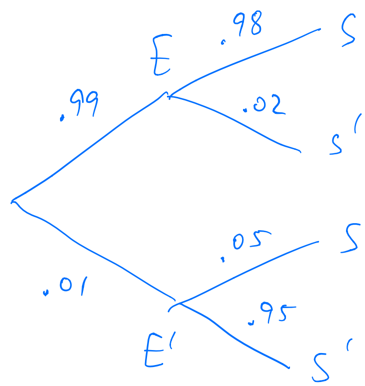
(a) Draw a tree diagram representing this situation. Explain your notation and be sure to include the probabilities.

E = Eagles fan

E' = not Eagles fan

S = own sweatshirt

S' = don't own sweatshirt



(b) If a person in Philadelphia is chosen at random, find the probability that she owns an Eagles sweatshirt. Show your work!!!

You want to wind up in one of the two endpoints marked S.

$(.99)(.98) + (.01)(.05) = .9707$

(c) A person in Philadelphia is chosen at random, and it is determined that he does not own an Eagles sweatshirt. Find the probability that he is an Eagles fan. Show your work!!!

$$P(E \mid S') = \frac{P(E \cap S')}{P(S')} = \frac{(.99)(.02)}{(.99)(.02) + (.01)(.95)} = .6758$$

(to 4 decimal places)

14. (10 pts.) A bag contains 9 colored marbles, of which 5 are red and 4 are blue. I plan to pick 3 marbles from the bag **without replacement**.

Note: In the following two parts, it is not necessary to give a numerical answer, i.e. you may express your answers using the notation for permutations ($P(n, k)$), combinations ($C(n, k)$), factorials ($n!$) and powers (a^k). Be sure to mark your answer.

(a) Assuming that order is not important, what is the probability that all three are the same color?

$$\frac{C(5,3)+C(4,3)}{C(9,3)} = \frac{10+4}{84} = \frac{1}{6}$$

(b) Assuming that order **is** important, what is the probability that the first is red, then the second is blue, then the third is red. [Don't forget that this is done **without** replacement.]

$$\left(\frac{5}{9}\right)\left(\frac{4}{8}\right)\left(\frac{4}{7}\right) = \frac{10}{63}$$

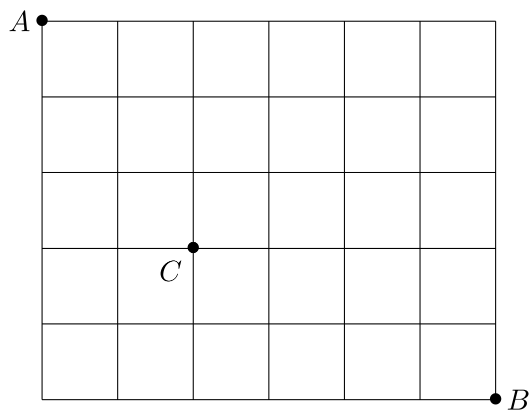
when you remove a red you have 4 red, 4 blue left. Then when you remove a blue you have 4 red and 3 blue left

15. (10 pts.) In this problem, be sure to show all your work and be sure to plainly mark your answer.

The following is a street map of part of a city. Emily lives at the northwest corner (marked A) and wants to get to the library at the southeast corner (marked B). She decides to take an Uber. The driver travels only east and south (i.e. to the right or down), following the roads and randomly choosing between east and south at each intersection he comes to. What is the probability that the Uber will pass by Claire's house, marked C?

[Hint: how many routes are there to get to the library, and of those how many pass by C?]

Note: In this problem, it is not necessary to give a numerical answer, i.e. you may express your answers using the notation for permutations ($P(n, k)$), combinations ($C(n, k)$), factorials ($n!$) and powers (a^k).



$$\frac{C(5,3)\,C(6,2)}{C(11,5)}$$

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MC.	________
11.	________
12.	________
13.	________
14.	________
15.	________
Tot.	________